# Mark Scheme (Results)

October 2020

Pearson Edexcel GCE Further Mathematics
Advanced Subsisiary Level
in Further Pure 1 Paper 8FM0_21

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## General Marking Guidance

- All candidates must receive the same treatment. Examiners must mark the last candidate in exactly the same way as they mark the first.
- Mark schemes should be applied positively. Candidates must be rewarded for what they have shown they can do rather than penalised for omissions.
- Examiners should mark according to the mark scheme not according to their perception of where the grade boundaries may lie.
- All the marks on the mark scheme are designed to be awarded. Examiners should always award full marks if deserved, i.e. if the answer matches the mark scheme. Examiners should also be prepared to award zero marks if the candidate's response is not worthy of credit according to the mark scheme.
- Where some judgement is required, mark schemes will provide the principles by which marks will be awarded and exemplification/indicative content will not be exhaustive.

**EDEXCEL GCE MATHEMATICS**

**General Instructions for Marking**

1. The total number of marks for the paper is 40.

2. The Edexcel Mathematics mark schemes use the following types of marks:
   - **M** marks: method marks are awarded for 'knowing a method and attempting to apply it', unless otherwise indicated.
   - **A** marks: Accuracy marks can only be awarded if the relevant method (M) marks have been earned.
   - **B** marks are unconditional accuracy marks (independent of M marks)
   - Marks should not be subdivided.

3. Abbreviations

These are some of the traditional marking abbreviations that will appear in the mark schemes.

- bod – benefit of doubt
- ft – follow through
- the symbol $\sqrt{}$ will be used for correct ft
- cao – correct answer only
- cso - correct solution only. There must be no errors in this part of the question to obtain this mark
- isw – ignore subsequent working
- awrt – answers which round to
- SC: special case
- oe – or equivalent (and appropriate)
- dep – dependent
- indep – independent
- dp decimal places
- sf significant figures
- ∗ The answer is printed on the paper
- ⌈ The second mark is dependent on gaining the first mark

4. For misreading which does not alter the character of a question or materially simplify it, deduct two from any A or B marks gained, in that part of the question affected.

5. Where a candidate has made multiple responses and indicates which response they wish to submit, examiners should mark this response. If there are several attempts at a question which have not been crossed out, examiners should mark the final answer which is the answer that is the most complete.

6. Ignore wrong working or incorrect statements following a correct answer.

7. Mark schemes will firstly show the solution judged to be the most common response expected from candidates. Where appropriate, alternatives answers are provided in the notes. If examiners are not sure if an answer is acceptable, they will check the mark scheme to see if an alternative answer is given for the method used.

## General Principles for Further Pure Mathematics Marking

(*But note that specific mark schemes may sometimes override these general principles*)

**<u>Method mark for solving 3 term quadratic:</u>**

**1. Factorisation**

$(x^2+bx+c)=(x+p)(x+q)$, where $|pq|=|c|$, leading to $x=\ldots$

$(ax^2+bx+c)=(mx+p)(nx+q)$, where $|pq|=|c|$ and $|mn|=|a|$, leading to $x=\ldots$

**2. Formula**

Attempt to use the correct formula (with values for *a*, *b* and *c*)

**3. Completing the square**

Solving $x^2+bx+c=0$ : $\left(x\pm\frac{b}{2}\right)^2\pm q\pm c=0,\quad q\neq 0$ , leading to $x=\ldots$

**<u>Method marks for differentiation and integration:</u>**

**1. Differentiation**

Power of at least one term decreased by 1. $(x^n\rightarrow x^{n-1})$

**2. Integration**

Power of at least one term increased by 1. $(x^n\rightarrow x^{n+1})$

**<u>Use of a formula</u>**

Where a method involves using a formula that has been learnt, the advice given in recent examiners' reports is that the formula should be quoted first.

Normal marking procedure is as follows:

<u>Method mark</u> for quoting a correct formula and attempting to use it, even if there are small errors in the substitution of values.

Where the formula is <u>not</u> quoted, the method mark can be gained by implication from <u>correct</u> working with values but may be lost if there is any mistake in the working.

**<u>Exact answers</u>**

Examiners' reports have emphasised that where, for example, an exact answer is asked for, or working with surds is clearly required, marks will normally be lost if the candidate resorts to using rounded decimals.

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 1 | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2y^2-x-1\Rightarrow\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0=0-0-1=-1$ | B1 | 1.1b |
| | $\left(\frac{\mathrm{d}y}{\mathrm{d}x}\right)_0=3\Rightarrow\frac{(y_1-y_{-1})}{0.2}\approx 3$ | B1 | 1.1b |
| | $\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_0\approx\frac{(y_1-2y_0+y_{-1})}{h^2}\Rightarrow\frac{y_1-2(0)+y_{-1}}{0.01}\approx -1$ | M1 | 1.1b |
| | $y_1\approx\frac{1}{2}(0.6-0.01)=0.295$ | dM1 | 2.1 |
| | $\frac{\mathrm{d}^2y}{\mathrm{d}x^2}=2y^2-x-1\Rightarrow\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_1=2(0.295)^2-0.1-1=-0.92595$ | dM1 | 1.1b |
| | $\left(\frac{\mathrm{d}^2y}{\mathrm{d}x^2}\right)_1\approx\frac{(y_2-2y_1+y_0)}{h^2}\Rightarrow\frac{y_2-2(0.295)+0}{0.01}\approx -0.92595\Rightarrow y_2=...$ | dM1 | 2.1 |
| | $y_2\approx 2(0.295)-0.92595\times 0.01=0.581$ (3 s.f.) | A1 | 1.1b |
| | | **(7)** | |
| | | | **(7 marks)** |

**Notes**

B1: Correct value for the second derivative using the differential equation
B1: Correct equation in terms of $y_1$ and $y_{-1}$ using the first order approximation
M1: Uses the second order approximation to obtain another equation in terms of $y_1$ and $y_{-1}$
M1: Uses their two equations in $y_1$ and $y_{-1}$ and solves together to find $y$ at $x$ = 0.1.
M1: Uses the differential equation with their $y$ at $x$ = 0.1 and $x$ = 0.1 to find a value for the second derivative at $x$ = 0.1
M1: Completes the process by using the second order approximation and their second derivative to obtain a value for $y_2$
A1: Correct value for $y$ at $x$ = 0.2
**Note that all method marks are dependent**

| Question | Scheme | Marks | AOs |
| --- | --- | --- | --- |
| 2 | $\frac{x+1}{2x^2+5x-3} > \frac{x}{4x^2-1}$ | | |
| | $\frac{2x^2+3x+1-x^2-3x}{(2x-1)(2x+1)(x+3)} > 0$ or $(x+1)(2x-1)(2x+1)^2(x+3) - x(2x-1)(2x+1)(x+3)^2 > 0$ | M1 | 2.1 |
| | $\frac{x^2+1}{(2x-1)(2x+1)(x+3)} > 0$ or $(x+3)(2x-1)(2x+1)(x^2+1) > 0$ | dM1 | 1.1b |
| | All three critical values $-3, -\frac{1}{2}, \frac{1}{2}$ | A1 | 1.1b |
| | $\left\{x \in \mathbb{R} : -3 < x < -\frac{1}{2}\right\} \cup \left\{x \in \mathbb{R} : x > \frac{1}{2}\right\}$ | dM1<br>A1 | 2.2a<br>2.5 |
| | | **(5)** | |
| | | | **(5 marks)** |

**Notes**

M1: Gathers terms on one side and puts over a common denominator, or multiplies by $(2x + 1)^2(2x - 1)^2(x + 3)^2$ and gathers terms on one side

dM1: Expands and simplifies numerator or factorises into 4 factors. Depends on the previous method mark.

A1: Correct critical values and no "extras" but ignore any attempts to solve $x^2 + 1 = 0$ (correct or otherwise)

dM1: Deduces that 1 "inside" inequality and 1 "outside" inequality is required with critical values in ascending order. Depends on the previous method mark.

A1: Exactly 2 correct intervals, accepting equivalent notation

**<u>Special Case:</u> Allow M1M0A0M0A0**

$$\frac{x+1}{2x^2+5x-3} > \frac{x}{4x^2-1} \Rightarrow \frac{x+1}{(2x-1)(x+3)} > \frac{x}{(2x-1)(2x+1)} \Rightarrow \frac{x+1}{(x+3)} > \frac{x}{(2x+1)}$$

$$\Rightarrow (x+1)(x+3)(2x+1)^2 > x(x+3)^2(2x+1) \text{ etc.}$$

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 3(i) | $\text{lhs} = \cot x + \tan\left(\frac{x}{2}\right) = \frac{1-t^2}{2t} + t$ | M1 | 1.1a |
| | $\frac{1-t^2}{2t} + t = \frac{1+t^2}{2t}\left(= \frac{1}{\sin x}\right) = \text{cosec } x*$ | A1* | 2.1 |
| | | **(2)** | |
| (ii)(a) | $x = 0 \Rightarrow H = 90 - 30\cos(0) - 40\sin(0) = 90 - 30 = 60$ | B1 | 1.1b |
| | | **(1)** | |
| (b) | $H = 90 - 30\cos 120x - 40\sin 120x = 90 - 30\left(\frac{1-t^2}{1+t^2}\right) - 40\left(\frac{2t}{1+t^2}\right)$ | M1 | 1.1b |
| | $= \frac{90 + 90t^2 - 30 + 30t^2 - 80t}{1+t^2}$ | M1 | 1.1b |
| | $= \frac{120t^2 - 80t + 60}{1+t^2}*$ | A1* | 2.1 |
| | | **(3)** | |
| (c) | $\frac{120t^2 - 80t + 60}{1+t^2} = 100 \Rightarrow 120t^2 - 80t + 60 = 100 + 100t^2$ | M1 | 3.4 |
| | $20t^2 - 80t - 40 = 0$ | A1 | 1.1b |
| | $t = \frac{4 \pm \sqrt{16+8}}{2} \Rightarrow 60x = \tan^{-1}\left(2+\sqrt{6}\right)$ or $60x = \tan^{-1}\left(2-\sqrt{6}\right)$ | M1 | 3.4 |
| | $60x = \tan^{-1}\left(2+\sqrt{6}\right) = 77.33... \Rightarrow x = ...$ | dM1 | 3.1b |
| | $x = 1.29$ | A1 | 3.2a |
| | | **(5)** | |
| | | | **(11 marks)** |

**Notes**

(ii)
M1: Selects the correct expression for cot $x$ in terms of $t$ and substitutes this and $t$ into the lhs
A1*: Fully correct proof. Allow correct work leading to $\frac{1+t^2}{2t} = \text{cosec } x$
(ii)(a)
B1: Demonstrates that when $x = 0$, $H = 60$
(b)
M1: Uses the correct formulae to obtain $H$ in terms of $t$
M1: Correct method to obtain a common denominator
A1*: Collects terms and simplifies to obtain the printed answer with no errors
(c)
M1: Uses $H = 100$ with the model and multiplies up to obtain a quadratic equation in $t$
A1: Correct 3TQ
M1: Solves their 3TQ in $t$ and proceeds to obtain values of $60x$ as suggested by the model
M1: A fully correct strategy to identify the required value of $x$ from the positive root of the quadratic equation in $t$
A1: awrt 1.29
Attempts in radians can score all but the final mark in (c). (Gives $60x = 1.3…$ etc.)

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| **4(a)** | $(a,0)$ | B1 | 1.1b |
| | | **(1)** | |
| **(b)** | $SP = ap^2 + a$<br>Note that if focus-directrix property not used may use Pythagoras:<br>E.g. $SP = \sqrt{4a^2p^2 + (ap^2 - a)^2} = ... = ap^2 + a$ | B1 | 1.1b |
| | | **(1)** | |
| **(c)** | $M$ has coordinates $\left(\frac{ap^2 + aq^2}{2}, \frac{2ap + 2aq}{2}\right)$ | B1 | 1.1b |
| | $y^2 = a^2\left(p^2 + 2pq + q^2\right)$ | M1 | 1.1b |
| | $y^2 = a^2\left(p^2 - 2 + q^2\right)$ | A1 | 2.1 |
| | $2a(x-a) = 2a\left(\frac{1}{2}ap^2 + \frac{1}{2}aq^2 - a\right) = a^2\left(p^2 + q^2 - 2\right)$ | M1 | 1.1b |
| | $\Rightarrow y^2 = 2a(x-a)$* | A1* | 2.1 |
| | | **(5)** | |
| | **Alternative for (c)** | | |
| | $M$ has coordinates $\left(\frac{ap^2 + aq^2}{2}, \frac{2ap + 2aq}{2}\right)$ | B1 | 1.1b |
| | $\frac{y}{a} = p + q$ | M1 | 1.1b |
| | $\frac{y^2}{a^2} = p^2 + q^2 + 2pq = p^2 + q^2 - 2$ | A1 | 2.1 |
| | $\frac{2x}{a} = p^2 + q^2$ | M1 | 1.1b |
| | $\frac{y^2}{a^2} = \frac{2x}{a} - 2 \Rightarrow y^2 = 2a\left(x-a\right)$* | A1* | 2.1 |
| | | **(5)** | |
| | | | **(7 marks)** |

| Notes |
| --- |
| (a)<br>B1: Correct coordinates<br>(b)<br>B1: Correct expression<br>(c)<br>B1: Correct coordinates for the midpoint<br>M1: Squares their y coordinate of the midpoint<br>A1: Uses $pq=-1$ to obtain a correct expression for $y^2$<br>M1: Attempts $2a(x-a)$ using the $x$ coordinate of their midpoint and attempts to simplify<br>A1*: Fully correct completion to show $y^2=2a(x-a)$<br>Alternative<br>B1: Correct coordinates for the midpoint<br>M1: Uses their $y$ coordinate of the midpoint to find $p+q$<br>A1: Square and uses $pq=-1$ to obtain a correct expression for $y^2/a^2$<br>M1: Uses the $x$ coordinate of their midpoint to find $p^2+q^2$<br>A1*: Fully correct completion to show $y^2=2a(x-a)$ |

| Question | Scheme | Marks | AOs |
|---|---|---|---|
| 5(a) | $\pm\overrightarrow{DE} = \pm\begin{pmatrix}-4\\3\\1\end{pmatrix},\ \pm\overrightarrow{DF} = \pm\begin{pmatrix}-1\\5\\2\end{pmatrix},\ \pm\overrightarrow{EF} = \pm\begin{pmatrix}3\\2\\1\end{pmatrix}$ | M1 | 1.1b |
| | Area = $\frac{1}{2}\left\|\overrightarrow{DE}\times\overrightarrow{DF}\right\| = \begin{vmatrix}\mathbf{i} & \mathbf{j} & \mathbf{k}\\-4 & 3 & 1\\-1 & 5 & 2\end{vmatrix} = \frac{1}{2}\begin{vmatrix}1\\7\\-17\end{vmatrix} = \frac{1}{2}\sqrt{1^2+7^2+17^2}$ | M1 | 1.1b |
| | $=\frac{1}{2}\sqrt{339}(\text{cm}^2)$ * | A1* | 2.2a |
| | | **(3)** | |
| | **Alternative for (a) using trigonometry:** | | |
| | $\pm\overrightarrow{DE} = \pm\begin{pmatrix}-4\\3\\1\end{pmatrix},\ \pm\overrightarrow{DF} = \pm\begin{pmatrix}-1\\5\\2\end{pmatrix},\ \pm\overrightarrow{EF} = \pm\begin{pmatrix}3\\2\\1\end{pmatrix}$ | M1 | 1.1b |
| | $DE=\sqrt{26},\ DF=\sqrt{30},\ EF=\sqrt{14}$<br>$\cos DEF = \frac{26+14-30}{2\sqrt{26}\sqrt{14}} \Rightarrow DEF = \cos^{-1}\frac{5}{\sqrt{26}\sqrt{14}}$<br>$\Rightarrow \text{Area } DEF = \frac{1}{2}\times\sqrt{26}\sqrt{14}\sqrt{1-\frac{25}{364}} = ...$ | M1 | 1.1b |
| | $=\frac{1}{2}\times\sqrt{26}\sqrt{14}\frac{\sqrt{339}}{\sqrt{26}\sqrt{14}} = \frac{1}{2}\sqrt{339}$ * | A1 | 2.2a |
| (b) | Attempt to find "$T$", the 4th vertex of the tetrahedron e.g.<br>$\begin{pmatrix}2\\-1\\8\end{pmatrix}+\lambda\overrightarrow{AD} = \begin{pmatrix}1\\4\\10\end{pmatrix}+\mu\overrightarrow{CF} \Rightarrow \lambda = ...$ or $\mu = ...$<br>or e.g.<br>$DF=\sqrt{30},\ AC=\sqrt{120} \Rightarrow$ Linear SF $=2$<br>$AT = 2AD \Rightarrow T$ is ... | M1 | 3.1b |
| | $T(1, 1, 15)$ | A1 | 1.1b |
| | e.g $\overrightarrow{AT} = \begin{pmatrix}-2\\4\\14\end{pmatrix},\ \overrightarrow{BT} = \begin{pmatrix}6\\-2\\12\end{pmatrix},\ \overrightarrow{CT} = \begin{pmatrix}0\\-6\\10\end{pmatrix}$<br>$\overrightarrow{AT}\times\overrightarrow{BT}\bullet\overrightarrow{CT} = \begin{vmatrix}-2 & 4 & 14\\6 & -2 & 12\\0 & -6 & 10\end{vmatrix} = ...$ | M1 | 1.1b |
| | e.g $V = \frac{1}{6}\left\|-2(-20+72)-4(60)+14(-36)\right\|\left(=\frac{424}{3}\right)$ | A1 | 1.1b |
| | $\frac{1}{6}\overrightarrow{DT}\times\overrightarrow{ET}\bullet\overrightarrow{FT} = \begin{vmatrix}-1 & 2 & 7\\3 & -1 & 6\\0 & -3 & 5\end{vmatrix} = \frac{1}{6}\left\|-1(13)-2(15)+7(-9)\right\|\left(=\frac{53}{3}\right)$ | M1 | 3.1b |

| | | | |
|---|---|---|---|
| | or<br>length scale factor = 2 $\Rightarrow$ volume scale factor = 8 | | |
| | e.g Volume = $\frac{424}{3}-\frac{53}{3}=...$ or Volume $=\frac{7}{8}\times\frac{424}{3}=...$<br>or Volume $=7\times\frac{53}{3}=...$ | dM1 | 3.1a |
| | $=\frac{371}{3}$ cm$^3$ | A1 | 1.1b |
| | | **(7)** | |
| | **See below for alternative for part (b)** | | |
| | | | **(10 marks)** |

**Notes**

(a)
M1: Attempts to find 2 edges of triangle *DEF*. Must be subtracting components.
M1: Uses the correct process of the vector product to attempt the area including use of Pythagoras
A1*: Deduces the correct area with no errors
Alternative:
M1: Attempts to find 3 edges of triangle *DEF*. Must be subtracting components.
M1: A complete method for the area. Allow work in decimals for this mark but must work in exact terms to obtain the A mark
A1*: Deduces the correct area with no errors and no decimal work
(b)
M1: Adopts a correct strategy to find the fourth vertex of the tetrahedron e.g. finding where two edges intersect or uses the linear scale factor
A1: Correct coordinates for the other vertex
M1: Uses the information from the design to attempt a scalar triple product between appropriate vectors to find the volume of the smaller or larger tetrahedron.
A1: Correct volume for either tetrahedron
M1: Makes further progress with the solution by finding the volume of the other tetrahedron or calculates the volume scale factor using an appropriate method. E.g. using ratios or by finding the area of triangle *DEF* and comparing with triangle *ABC*
dM1: Completes the problem by finding the required volume of the frustum. Depends on all previous method marks
A1: Correct answer

**Alternative for part (b) – splits into 4 tetrahedra:**
**This example takes *M* as the midpoint of *AC* and finds the volume of *ABMD*, *MBFC*, *DMFB*, *EDFB***

| | | | |
|---|---|---|---|
| **(b)** | $Vol_{ABMD}=\frac{1}{6}\overrightarrow{AB}\times\overrightarrow{AD}\bullet\overrightarrow{AM}=...$ | M1 | 3.1b |
| | $=\frac{106}{3}$ | A1 | 1.1b |
| | $Vol_{MBFC}=\frac{1}{6}\overrightarrow{BF}\times\overrightarrow{BC}\bullet\overrightarrow{BM}=...\left(\frac{106}{3}\right)$<br>$Vol_{DMFB}=\frac{1}{6}\overrightarrow{DF}\times\overrightarrow{DM}\bullet\overrightarrow{DB}=...\left(\frac{106}{3}\right)$ | M1<br>A1 | 1.1b<br>1.1b |

| | | | |
|---|---|---|---|
| | $Vol_{EDFB} = \frac{1}{6}\overrightarrow{ED}\times\overrightarrow{EF}\bullet\overrightarrow{EB} = ...\left(\frac{53}{3}\right)$ | M1 | 3.1b |
| | *ABMD* + *MBFC* + *DMFB* + *EDFB* = $3\times\frac{106}{3}+\frac{53}{3}$ | dM1 | 3.1a |
| | $=\frac{371}{3}\text{ cm}^3$ | A1 | 1.1b |

**Notes:**

M1: Adopts a correct strategy to find the volume of one tetrahedron
A1: Any correct volume
M1: Adopts a correct strategy to find the volumes of at least 2 other tetrahedra
A1: Correct volumes
M1: Makes further progress with the solution by finding the volume of all relevant tetrahedra
dM1: Completes the problem by finding the required volume of the frustum. Depends on all previous method marks
A1: Correct answer